EXERCISES 10.1 (submit by 17.06.2016)

1. In each of the cases below, find the limit $f(x)$ of $f_n(x)$ for $x \in [0,1]$. Does f_n converge to f uniformly on $[0,1]$?

   (a) $f_n(x) = \frac{1}{1+nx}$
   (b) $f_n(x) = \frac{x}{1+n^2x^2}$
   (c) $f_n(x) = \frac{nx}{1+n^2x^2}$
   (d) $f_n(x) = n^2 x e^{-n^2x^2}$
   (e) $f_n(x) = x e^{-n^2x^2}$.

2. Which of the following series converge uniformly on $\mathbb{R}$? Which of the sums are continuous functions of x on $\mathbb{R}$?

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^2}{(1+x^2)^n}$
   (b) $\sum_{n=1}^{\infty} \frac{x^2}{(1+x^2)^n}$
   (c) $\sum_{n=1}^{\infty} \frac{1}{x^2+n^2}$
   (d) $\sum_{n=1}^{\infty} \frac{\sin nx}{x^2+n}$.

3. For which of the following series $\sum_{n=1}^{\infty} u_n(x)$ on $(0,\pi)$, the derivative of the sum equals to the sum of the derivatives $u_n'(x)$?

   (a) $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$
   (b) $\sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$
   (c) $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$.