Exercises 9.2, Mathematics 1 (12-PHY-BIPMA1)
Artem Sapozhnikov

1. Identify all $\alpha > 0$ for which the following series converge:

$$\text{(a)} \sum_{n=2}^{\infty} \frac{1}{n \ln^\alpha n} \ , \qquad \text{(b)} \sum_{n=2}^{\infty} \frac{1}{n^2 \ln^\alpha n} \ , \qquad \text{(c)} \sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln^\alpha n} \ .$$

2. Which of the following series converge?

$$\text{(a)} \sum_{n=1}^{\infty} \frac{1}{2^{\sqrt{n}}} \ , \qquad \text{(b)} \sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}} \ , \qquad \text{(c)} \sum_{n=1}^{\infty} \frac{2^n}{n!} \ , \qquad \text{(d)} \sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)^{n^2} \ ,$$

$$\text{(e)} \sum_{n=1}^{\infty} \left(\sqrt[n]{a} - 1 \right) \quad \text{for } a > 0, \qquad \text{(f)} \sum_{n=1}^{\infty} \ln \left(\cos \frac{1}{n} \right) \ .$$

3. Which of the following series converge? Which converge absolutely?

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \ , \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{\sin n}{n^2} \ , \qquad \text{(c)} \sum_{n=2}^{\infty} \frac{\cos n}{\ln n} \ .$$