Exercises 9.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Compute the improper integrals:

$$\text{(a)} \int_2^{+\infty} \frac{dx}{(x-1)^2}, \qquad \text{(b)} \int_0^1 \frac{dx}{\sqrt{1-x}}, \qquad \text{(c)} \int_0^{+\infty} e^{-x} dx, \qquad \text{(d)} \int_e^{+\infty} \frac{dx}{x(\ln x)^2}.$$

2. Compute the improper integrals $I_n = \int_0^{+\infty} x^n e^{-x} dx$, $n \geq 0$.

3. Identify all $\alpha > 0$ for which the following improper integrals converge:

$$\text{(a)} \int_2^{+\infty} \frac{dx}{x(\ln x)^\alpha}, \qquad \text{(b)} \int_2^{+\infty} \frac{dx}{x^2(\ln x)^\alpha}, \qquad \text{(c)} \int_2^{+\infty} \frac{dx}{\sqrt{x}(\ln x)^\alpha},$$

$$\text{(d)} \int_1^{+\infty} \frac{\sin x}{x^\alpha} dx, \qquad \text{(e)} \int_1^{+\infty} \frac{\cos x}{x^\alpha} dx, \qquad \text{(f)} \int_1^{+\infty} \frac{\ln x}{x^\alpha} dx,$$

$$\text{(g)} \int_1^2 \frac{dx}{(\ln x)^\alpha}, \qquad \text{(h)} \int_0^1 \frac{\sin x}{x^\alpha} dx.$$