

Exercises 6.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Use l'Hopital's rule to compute the limits:

(a) $\lim_{x \rightarrow a} \frac{a^x - x^a}{x - a}$, for $a > 0$,

(b) $\lim_{x \rightarrow 0+} x^\varepsilon \ln x$, for $\varepsilon > 0$,

(c) $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$,

(d) $\lim_{x \rightarrow 0} \left(\cot x - \frac{1}{x} \right)$,

(e) $\lim_{x \rightarrow e} \frac{\ln(\ln x)}{\sin(x-e)}$,

(f) $\lim_{x \rightarrow +\infty} \sqrt{x} \left(e^{-\frac{1}{x}} - 1 \right)$.

2. Consider the function

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Compute $f^{(n)}(0)$ for all $n \in \mathbb{N}$.