

Exercises 5.2, Mathematics 1 (12-PHY-BIPMA1)
Artem Sapozhnikov

1. Identify the intervals on which the following functions are monotone increasing:

$$f(x) = x^2 - x, \quad g(x) = \sin x, \quad h(x) = \frac{x}{1+x^2}, \quad k(x) = \arcsin x.$$

2. Find local maxima and minima (if any) of the following functions:

$$f(x) = x^3 + 2x^2, \quad g(x) = x^5 - 2x^4, \quad h(x) = 1 - |x|, \quad k(x) = \sin^2 x.$$

3. Find the global maximum and minimum of the following functions on the specified intervals:

$$f(x) = x^3 - 3x^2, \quad x \in [-1, 4], \quad g(x) = \sin x + \cos^2 x, \quad x \in [0, \pi].$$

4. Let $a \in \mathbb{R}$ and $x \geq -1$, $x \neq 0$. Use Lagrange's theorem to prove the following inequalities:

- (a) if $0 < a < 1$ then $(1+x)^a < 1+ax$,
(b) if $a < 0$ or $a > 1$ then $(1+x)^a > 1+ax$.

Hint: Apply Lagrange's theorem to function $f(z) = (1+z)^a$ on the interval $[0, x]$ (if $x > 0$) or $[x, 0]$ (if $x < 0$).