Exercises 3.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Use the $\varepsilon - \delta$ definition to prove the following limits:
   (a) $\lim_{x\to 1} 3x + 1 = 4$,
   (b) $\lim_{x\to 2} x^3 = 8$,
   (c) $\lim_{x\to 9} \sqrt{x} = 3$,
   (d) $\lim_{x\to 5} \frac{3+x}{1+3x} = \frac{1}{2}$,
   (e) $\lim_{x\to -1+} \sqrt{x+1} = 0$.
2. Using properties of the limits, compute the following:
   (a) $\lim_{x\to 1} \frac{x^2-x-2}{x^2-1}$,
   (b) $\lim_{x\to \infty} \frac{x^2-x-2}{x^2-1}$,
   (c) $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$.
3. Find $a \in \mathbb{R}$ such that the function
$$f(x) = \begin{cases} 3x+2 & \text{for } x < 2 \\ x^2 + a & \text{for } x \geq 2 \end{cases}$$
   is continuous. (Hint: Compute left and right limits.)
4. Prove that there exists $x \in [1, 2]$ such that $2^x = \pi$. (Hint: Use the continuity of $f(x) = 2^x$.)
5. Prove that $\lim_{x\to\infty} \frac{x^2}{e^x} = 0$. (Hint: Use the fact that $2^n \geq n^3$ for all $n \geq 10$ to prove that $e^x \geq (x-1)^3$ for all $x \geq 10$.)