Exercises 2.2, Mathematics 1 (12-PHY-BIPMA1)
Artem Sapozhnikov

1. Find the domain and the range of the following functions:
   (a) $f(x) = \frac{1}{1+x}$,
   (b) $f(x) = \sqrt{x} - \sqrt{x-1}$,
   (c) $f(x) = \frac{1}{\sqrt{1+x^2}}$,
   (d) $f(x) = \sin x$,
   (e) $f(x) = \tan x$.

2. Find formulas for the following implicitly defined functions. What are their domains and ranges?
   (a) $y = f(x)$ is the solution of equation $x^3y + 2y = 5$,
   (b) $y = f(x)$ is the largest solution of equation $y^2 = 3x^2 - 2xy$,
   (c) $y = f(x)$ is the solution of equation $2x + 2xy + y^2 = 5$ which satisfies $y > -x$.

3. A function f is given which satisfies $f(2x + 3) = x^2$ for all $x \in \mathbb{R}$. Compute
   (a) $f(0)$,
   (b) $f(3)$,
   (c) $f(x)$,
   (d) $f(y)$,
   (e) $f(f(2))$.

4. What is the limit of the following sequence?
$$\lim_{n\to\infty} 8^{\frac{n+1}{3n+2}}.$$

5. Which of the following limits exist?
   (a)
$$\lim_{x\to 1} \frac{x^2 - 3x + 2}{x^2 - 1},$$
   (b)
$$\lim_{x\to 0} \sin \frac{1}{x^2},$$
   (c)
$$\lim_{x\to -1} |\mathrm{sign}(x + 1)|.$$