## Exercises 1.1, Mathematics 1 (12-PHY-BIPMA1) Artem Sapozhnikov

- 1. Write the following sets as unions of intervals:
  - (a)  $A = \{x : x^2 3x + 2 \le 0\},\$ (b)  $B = \{x : x^2 - 3x + 2 \ge 0\},\$ (c)  $C = \{x : x^2 - 3x > 3\},\$ (d)  $D = \{x : x^2 - 5 > 2x\},\$ (e)  $E = \{t : t^2 - 3t + 2 \le 0\},\$ (f)  $F = (\{1\} \cup \{2,3\}) \cap (0,4),\$ (g)  $G = \{\theta : \sin \theta = \frac{1}{2}\},\$ (h)  $H = \{\varphi : \cos \varphi > 0\}.\$
- 2. Let A and B be intervals. Is  $A \cap B$  an interval? What about  $A \cup B$ ?
- 3. Prove that the following numbers are rational:
  - (a) x = 0.313131...,(b) y = 0,273273273...,(c) z = 0.2154154154....

(Hint: Note that 100x = x + 31.)

4. Let A and B be two sets. Define their sum and difference as

 $A + B = \{a + b : a \in A, b \in B\}, \qquad A - B = \{a - b : a \in A, b \in B\}.$ 

Prove that  $\sup(A + B) = \sup A + \sup B$ . Prove that  $\sup(A - B) = \sup A - \inf B$ .

- 5. Prove that  $\mathbb{Q}$  is countable.
- 6. Let  $A \subseteq B$ . Prove that if A is uncountable, then B is also uncountable.