

Exercises 10.2, Mathematics 1 (12-PHY-BIPMA1)
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1. Compute the real and imaginary parts of

$$(a) (1+i)^2, \quad (b) (1-2i)^3, \quad (c) \frac{1+i}{1-i}, \quad (d) i^n, \text{ for all } n \in \mathbb{Z}.$$

2. Find $x, y \in \mathbb{R}$ such that

$$(1+2i)x + (3-i)y = 5+3i.$$

3. Find loci of the points in complex plane such that

$$(a) |z-i| \leq 1, \quad (b) |z-1| = |z-i|.$$

4. For which $z_1, z_2 \in \mathbb{C}$,

$$|z_1 + z_2| = |z_1| + |z_2|.$$

5. Use the trigonometric form of complex numbers to compute

$$(1-i\sqrt{3})^3(1+i)^2.$$

6. Find

$$(a) \sqrt[3]{i}, \quad (b) \sqrt[4]{-4}, \quad (c) \sqrt[6]{1}, \quad (d) \sqrt[3]{2-2i}.$$

7. Prove that

$$\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11} = \frac{1}{2}.$$

[Hint: Use Euler's formula.]

8. Let $x \in \mathbb{R}$. Compute the limits

$$(a) \lim_{n \rightarrow \infty} \frac{e^{inx}}{2^n}, \quad (b) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} \cos x + \dots + \frac{1}{2^n} \cos nx \right).$$

[Hint: Use Euler's formula in (b).]