

Exercises 10.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Which of the following series of functions converge uniformly on specified intervals?

$$(a) \sum_{n=1}^{\infty} \frac{x^n}{n^2}, x \in [-1, 1], \quad (b) \sum_{n=1}^{\infty} \frac{1}{n^{2+x}}, x \geq 0, \quad (c) \sum_{n=1}^{\infty} \frac{x^n}{n}, x \in \left[0, \frac{1}{2}\right],$$

$$(d) \sum_{n=1}^{\infty} \frac{x^n}{n}, x \in [0, 1), \quad (e) \sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}, x \in \mathbb{R}, \quad (f) \sum_{n=1}^{\infty} xe^{-n^2x}, x \geq 0.$$

2. Compute the following sums:

$$(a) \sum_{n=1}^{\infty} \frac{1}{n2^n}, \quad (b) \sum_{n=1}^{\infty} \frac{n^2}{2^n}.$$

3. Identify the radii of convergence of the following series:

$$(a) \sum_{n=1}^{\infty} n^2 x^n, \quad (b) \sum_{n=1}^{\infty} \frac{x^n}{n^3}, \quad (c) \sum_{n=1}^{\infty} \left(\frac{x}{n}\right)^n.$$