

Exercises 9.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Compute the following integrals:

$$(a) \int_1^2 x^2 dx, \quad (b) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x dx, \quad (c) \int_0^1 \frac{dx}{\sqrt{1-x^2}}, \quad (d) \int_1^5 \frac{dx}{x}.$$

2. Use substitutions to compute the following integrals:

$$(a) \int_0^2 \frac{x dx}{1+x^2}, \quad (b) \int_0^\pi \cos(\varphi + \frac{\pi}{3}) d\varphi, \quad (c) \int_{-2}^0 \frac{dx}{x^2 + 2x + 2}, \quad (d) \int_0^{\frac{\pi}{6}} \frac{dx}{\cos x}.$$

3. Use integration by parts to compute the following integrals:

$$(a) \int_1^e (\ln x)^2 dx, \quad (b) \int_0^5 x e^x dx, \quad (c) \int_\pi^{2\pi} x \cos x dx, \quad (d) \int_0^{\frac{\pi}{4}} \frac{x dx}{\cos^2 x}.$$

4. Compute $\int_{-1}^1 f(x) dx$, where

$$f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } x \geq 0. \end{cases}$$

5. Find $f(x)$ such that $f'(x) = \sin^2 x$ and $f(\frac{\pi}{2}) = \frac{\pi}{4}$.