Exercises 6.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Compute the derivatives of the following functions (at the points where they exist):

   (a) $f(x) = \sin(\ln x)$,

   (b) $f(x) = \ln(1 + \sin x)$,

   (c) $f(x) = x^x$,

   (d) $f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}$,

   (e) $f(x) = \arctan\left(\frac{x+1}{x-1}\right)$.

2. Find general formulas for derivatives of the following functions for all $n \in \mathbb{N}$:

   (a) $\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)^{(n)}$,

   (b) $(x^n e^x)^{(n)}$,

   (c) $\left(\frac{\ln x}{x}\right)^{(n)}$.

3. Compute the first and the second derivatives of the following function for each $x \in \mathbb{R}$:
$$f(x) = \begin{cases} x^4 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

   Notice that both derivatives change sign infinitely often to the left and to the right near 0.

4. Identify the intervals on which the following functions are monotone increasing:
$$f(x) = x^2 - x, \quad g(x) = \sin x, \quad h(x) = \frac{x}{1 + x^2}, \quad k(x) = \arcsin x.$$

5. Find local maxima and minima (if any) of the following functions:
$$f(x) = x^3 + 2x^2, \quad g(x) = x^5 - 2x^4, \quad h(x) = 1 - |x|, \quad k(x) = \sin^2 x.$$

6. Find the global maximum and minimum of the following functions on the specified intervals:
$$f(x) = x^3 - 3x^2, \ x \in [-1, 4], \quad g(x) = \sin x + \cos^2 x, \ x \in [0, \pi].$$