Exercises 5.2, Mathematics 1 (12-PHY-BIPMA1) Artem Sapozhnikov (submit by 20.11.2015)

1. Which of the following functions is differentiable at 0:

$$f(x) = x|x|, \quad g(x) = x\sqrt{|x|}, \quad h(x) = x + |x|,$$
$$k(x) = \begin{cases} x^2 \sin\frac{\pi}{x} & \text{if } x \neq 0\\ 0 & \text{if } x = 0, \end{cases} \quad l(x) = \begin{cases} x \sin\frac{\pi}{x} & \text{if } x \neq 0\\ 0 & \text{if } x = 0. \end{cases}$$

2. For which real numbers a and b, the following function is differentiable at 0:

$$f(x) = \begin{cases} ax+b & \text{for } x < 0\\ x-x^2 & \text{for } x \ge 0. \end{cases}$$

3. For which real numbers a and b, the following function is differentiable at 0:

$$f(x) = \begin{cases} ax+b & \text{for } x < 1\\ x-x^2 & \text{for } x \ge 1. \end{cases}$$

4. Compute the derivatives of the following functions:

$$f(x) = x \sin x + \cos x, \quad g(x) = \cos^2 x, \quad h(x) = \frac{\cos^2 x}{x},$$
$$k(x) = \frac{5e^x}{1 + \tan x}, \quad l(x) = \frac{1 + e^x}{1 - e^x}, \quad m(x) = e^x \sin x \cos^2 x.$$