

Exercises 4.2, Mathematics 1 (12-PHY-BIPMA1)
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1. Compute the limits:

(a) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{2x},$

(b) $\lim_{x \rightarrow 0} \frac{\arcsin(x^2)}{x^2},$

(c) $\lim_{x \rightarrow \infty} x \sin \frac{1}{x},$

(d) $\lim_{x \rightarrow 0} \cos(x^2 - \pi),$

(e) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2},$ (Hint: Multiply and divide by $1 + \cos x$, use $\sin^2 x + \cos^2 x = 1$.)

(f) $\lim_{y \rightarrow 0} \frac{1 - \cos y}{y \sin y},$

(g) $\lim_{x \rightarrow 0} \frac{\tan^2 x}{1 - \cos x},$

(h) $\lim_{z \rightarrow 0} \frac{\arctan(z^2)}{1 - \cos z}.$

2. Prove that the following functions are continuous:

(a)

$$f(x) = \begin{cases} \frac{\sin(1+x)}{1+x} & \text{for } x \neq -1, \\ 1 & \text{for } x = -1. \end{cases}$$

(b)

$$f(u) = \begin{cases} \frac{1 - \cos(\sin u)}{1 - \cos^2 u} & \text{for } u \neq 0, \pm\pi, \pm2\pi, \pm3\pi, \dots, \\ \frac{1}{2} & \text{for } u = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots \end{cases}$$