Exercises 4.1, Mathematics 1 (12-PHY-BIPMA1) Artem Sapozhnikov (submit by 13.11.2015)

1. Find $a \in \mathbb{R}$ such that the function

$$f(x) = \begin{cases} 3x+2 & \text{for } x < 2\\ x^2+a & \text{for } x \ge 2 \end{cases}$$

is continuous. (Hint: Compute left and right limits.)

- 2. For each a > 0, prove that the exponential function $f(x) = a^x$ is continuous on \mathbb{R} . (Hint: Note that it suffices to prove continuity at 0.)
- 3. Prove that there exists $x \in [1, 2]$ such that $2^x = \pi$. (Hint: Use the continuity of $f(x) = 2^x$.)