

Exercises 3.2, Mathematics 1 (12-PHY-BIPMA1)
Artem Sapozhnikov
(submit by 06.11.2015)

1. Which of the following limits exist?

(a)

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 1},$$

(b)

$$\lim_{x \rightarrow 0} \sin \frac{1}{x^2},$$

(c)

$$\lim_{x \rightarrow -1} |\operatorname{sign}(x + 1)|.$$

2. Use the $\varepsilon - \delta$ definition to prove the following limits:

(a) $\lim_{x \rightarrow 1} 3x + 1 = 4,$

(b) $\lim_{x \rightarrow 2} x^3 = 8,$

(c) $\lim_{x \rightarrow 9} \sqrt{x} = 3,$

(d) $\lim_{x \rightarrow 5} \frac{3+x}{1+3x} = \frac{1}{2},$

(e) $\lim_{x \rightarrow -1+} \sqrt{x+1} = 0.$

3. Using properties of the limits, compute the following:

(a) $\lim_{x \rightarrow 1} \frac{x^2 - x - 2}{x^2 - 1},$

(b) $\lim_{x \rightarrow \infty} \frac{x^2 - x - 2}{x^2 - 1},$

(c) $\lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}.$

4. Prove that $\lim_{x \rightarrow \infty} \frac{x^2}{e^x} = 0$. (Hint: Use the fact that $2^n \geq n^3$ for all $n \geq 10$ to prove that $e^x \geq (x - 1)^3$ for all $x \geq 10$.)