Exercises 13.1, Mathematics 1 (12-PHY-BIPMA1)
Artem Sapozhnikov
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1. Prove that any 4 vectors in the space are linearly dependent.

2. Let e_1, e_2, e_3 be the right-hand oriented basis. What is the orientation of (a) e_1, e_3, e_2, (b) e_2, e_1, e_3, (c) e_2, e_3, e_1, (d) e_3, e_1, e_2, (e) e_3, e_2, e_1?

3. Let e_1, e_2, e_3 be an orthonormal basis with right-hand orientation. If u and v are vectors with coordinates $(1, 2, 1)$ and $(1, 1, 0)$, respectively, what are the coordinates of $u \times v$?

4. Let e_1, e_2, e_3 be an orthonormal basis with right-hand orientation. Let $a \in \mathbb{R}$. Let u, v, w be vectors with coordinates $(1, 2, 1)$, $(1, 1, 0)$, and $(a, 1, 1)$, respectively. Identify all a for which u, v, w is (a) a basis, (b) a right-hand oriented basis.

5. Prove that
$$u \times (v \times w) = (u, w)v - (u, v)w.$$
[Hint: Consider a right-hand oriented orthonormal basis e_1, e_2, e_3 such that (a) the coordinates of w are $(w_1, 0, 0)$, i.e., e_1 is collinear with w, (b) the coordinates of v are $(v_1, v_2, 0)$, i.e., e_2 is coplanar with v and w, and (c) the coordinates of u are (u_1, u_2, u_3). Compute coordinates of $u \times (v \times w)$ and $(u, w)v - (u, v)w$ in this basis.]

6. Prove the Jacobi identity:
$$u \times (v \times w) + v \times (w \times u) + w \times (u \times v) = 0.$$