

Exercises 12.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Compute the real and imaginary parts of

(a) $(1+i)^2$, (b) $(1-2i)^3$, (c) $\frac{1+i}{1-i}$, (d) i^n , for all $n \in \mathbb{Z}$.

2. Find $x, y \in \mathbb{R}$ such that

$$(1+2i)x + (3-i)y = 5+3i.$$

3. Find loci of the points in complex plane such that

(a) $|z-i| \leq 1$, (b) $|z-1| = |z-i|$.

4. For which $z_1, z_2 \in \mathbb{C}$,

$$|z_1 + z_2| = |z_1| + |z_2|.$$

5. Use the trigonometric form of complex numbers to compute

$$(1-i\sqrt{3})^3(1+i)^2.$$

6. Find

(a) $\sqrt[3]{i}$, (b) $\sqrt[4]{-4}$, (c) $\sqrt[6]{1}$, (d) $\sqrt[3]{2-2i}$.

7. Prove that

$$\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11} = \frac{1}{2}.$$

[Hint: Use Euler's formula.]

8. Let $x \in \mathbb{R}$. Compute the limits

(a) $\lim_{n \rightarrow \infty} \frac{e^{inx}}{2^n}$, (b) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} \cos x + \dots + \frac{1}{2^n} \cos nx \right).$

[Hint: Use Euler's formula in (b).]