Exercises 1.2, Mathematics 1 (12-PHY-BIPMA1)
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1. Let $\lim_{n\to\infty} a_n = a$. Prove that $\lim_{n\to\infty} |a_n| = |a|$.

2. Let $\lim_{n\to\infty} a_n = a$. Let (b_n) be a sequence satisfying $b_{n+k} = a_{n+l}$ for some $k, l \in \mathbb{N}$ and all $n \in \mathbb{N}$. Prove that $\lim_{n\to\infty} b_n = a$.

3. Let $\lim_{n\to\infty} a_n = a$. Prove that $\lim_{n\to\infty} \sqrt{a_n} = \sqrt{a}$.

4. Prove using the definition of the limit that
   (a) $\lim_{n\to\infty} \frac{(-1)^n}{n^2} = 0$,
   (b) $\lim_{m\to\infty} \cos\frac{1}{m} = 1$,
   (c) $\lim_{k\to\infty} (-1)^k$ does not exist.