Exercises 11.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Which of the following series of functions converge uniformly on specified intervals?

$$\text{(a)} \sum_{n=1}^{\infty} \frac{x^n}{n^2} \ , x \in [-1, 1], \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{1}{n^{2+x}} \ , x \geq 0, \qquad \text{(c)} \sum_{n=1}^{\infty} \frac{x^n}{n} \ , x \in \left[0, \frac{1}{2}\right],$$

$$\text{(d)} \sum_{n=1}^{\infty} \frac{x^n}{n} \ , x \in [0, 1), \qquad \text{(e)} \sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)} \ , x \in \mathbb{R}, \qquad \text{(f)} \sum_{n=1}^{\infty} xe^{-n^2 x} \ , x \geq 0.$$

2. Compute the following sums:

$$\text{(a)} \sum_{n=1}^{\infty} \frac{1}{n2^n} \ , \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{n^2}{2^n} \ .$$

3. Identify the radii of convergence of the following series:

$$\text{(a)} \sum_{n=1}^{\infty} n^2 x^n \ , \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{x^n}{n^3} \ , \qquad \text{(c)} \sum_{n=1}^{\infty} \left(\frac{x}{n}\right)^n \ .$$