Exercises 1.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Write the following sets as unions of intervals:

   (a) $A = \{x : x^2 - 3x + 2 \leq 0\}$,
   (b) $B = \{x : x^2 - 3x + 2 \geq 0\}$,
   (c) $C = \{x : x^2 - 3x > 3\}$,
   (d) $D = \{x : x^2 - 5 > 2x\}$,
   (e) $E = \{t : t^2 - 3t + 2 \leq 0\}$,
   (f) $F = (\{1\} \cup \{2, 3\}) \cap (0, 4)$,
   (g) $G = \{\theta : \sin\theta = \frac{1}{2}\}$,
   (h) $H = \{\varphi : \cos\varphi > 0\}$.

2. Let A and B be intervals. Is $A \cap B$ an interval? What about $A \cup B$?

3. Prove that the following numbers are rational:

   (a) $x = 0.313131\ldots$,
   (b) $y = 0,273273273\ldots$,
   (c) $z = 0.2154154154\ldots$.

   (Hint: Note that $100x = x + 31$.)

4. Let A and B be two sets. Define their sum and difference as

$$A + B = \{a + b : a \in A, b \in B\}, \qquad A - B = \{a - b : a \in A, b \in B\}.$$

   Prove that $\sup(A + B) = \sup A + \sup B$. Prove that $\sup(A - B) = \sup A - \inf B$.

5. Prove that $\mathbb{Q}$ is countable.

6. Let $A \subseteq B$. Prove that if A is uncountable, then B is also uncountable.