

Exercises 10.2, Mathematics 1 (12-PHY-BIPMA1)
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1. Identify all $\alpha > 0$ for which the following series converge:

$$(a) \sum_{n=2}^{\infty} \frac{1}{n \ln^{\alpha} n}, \quad (b) \sum_{n=2}^{\infty} \frac{1}{n^2 \ln^{\alpha} n}, \quad (c) \sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln^{\alpha} n}.$$

2. Which of the following series converge?

$$(a) \sum_{n=1}^{\infty} \frac{1}{2^{\sqrt{n}}}, \quad (b) \sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}, \quad (c) \sum_{n=1}^{\infty} \frac{2^n}{n!}, \quad (d) \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2},$$
$$(e) \sum_{n=1}^{\infty} (\sqrt[n]{a} - 1) \quad \text{for } a > 0, \quad (f) \sum_{n=1}^{\infty} \ln \left(\cos \frac{1}{n}\right).$$

3. Which of the following series converge? Which converge absolutely?

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}, \quad (b) \sum_{n=1}^{\infty} \frac{\sin n}{n^2}, \quad (c) \sum_{n=2}^{\infty} \frac{\cos n}{\ln n}.$$

(A series $\sum_{n=1}^{\infty} u_n$ converges absolutely if the series $\sum_{n=1}^{\infty} |u_n|$ converges.)