Exercises 10.1, Mathematics 1 (12-PHY-BIPMA1)
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1. Find the area bounded by the curve $y = x(2 - x)$ and the line $y = \frac{1}{2}x$.

2. Find the volume of the solid of the revolution of the curve $y = \sin x$, $x \in [0, \pi]$ around x-axis.

3. Find the length of the parabola $y = x^2$, $x \in [0, 1]$.

4. The cycloid is a curve defined by $x(t) = a(t - \sin t)$ and $y(t) = a(1 - \cos t)$, $t \in [0, 2\pi]$. Find the area bounded by the cycloid and the x-axis. Find the length of the cycloid.

   [Hint: To compute the area, use the formula $\int_0^{2\pi} y dx$, where $dx = x'(t)dt$.]

5. Compute the improper integrals:

   (a) $\displaystyle\int_2^{+\infty} \frac{dx}{(x-1)^2}$,　　(b) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x}}$,　　(c) $\displaystyle\int_0^{+\infty} e^{-x}dx$,　　(d) $\displaystyle\int_e^{+\infty} \frac{dx}{x(\ln x)^2}$.

6. Identify all $\alpha > 0$ for which the following improper integrals converge:

   (a) $\displaystyle\int_2^{+\infty} \frac{dx}{x(\ln x)^\alpha}$,　　(b) $\displaystyle\int_2^{+\infty} \frac{dx}{x^2(\ln x)^\alpha}$,　　(c) $\displaystyle\int_2^{+\infty} \frac{dx}{\sqrt{x}(\ln x)^\alpha}$,

   (d) $\displaystyle\int_1^{+\infty} \frac{\sin x}{x^\alpha}dx$,　　(e) $\displaystyle\int_1^{+\infty} \frac{\cos x}{x^\alpha}dx$,　　(f) $\displaystyle\int_1^{+\infty} \frac{\ln x}{x^\alpha}dx$,

   (g) $\displaystyle\int_1^2 \frac{dx}{(\ln x)^\alpha}$,　　(h) $\displaystyle\int_0^1 \frac{\sin x}{x^\alpha}dx$.