Homework 2 (submit by 20.04.2017)

1. Prove that any continuous function $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue measurable.
2. Prove that any increasing function $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue measurable.
3. Consider a sequence of Lebesgue measurable functions $f_n : \mathbb{R} \to \mathbb{R}$. Prove that $f(x) = \limsup_{n\to\infty} f_n(x)$ is Lebesgue measurable.
4. Let f be a Lebesgue measurable function. Let $g(x) = f(x)$ almost everywhere. Prove that g is Lebesgue measurable.
5. Prove that the Borel σ-algebra on $\mathbb{R}$ is generated by each of the following families of subsets of $\mathbb{R}$:
   (a) $\{(-\infty, a],\ a \in \mathbb{R}\}$,
   (b) $\{(a, b],\ a, b \in \mathbb{R}\}$,
   (c) $\{[a, b],\ a, b \in \mathbb{R}\}$.