Homework 11 (submit by 22.06.2017)

1. Consider the surface $r(u, v) = (u \sin v, u \cos v, v)$ in $\mathbb{R}^3$. Find
   (a) the first fundamental form
   (b) the area of the triangle on the surface defined by $0 \le v \le v_0$, $0 \le u \le \sinh v$
   (c) the lengths of the sides of the triangle
   (d) the angles of the triangle.

2. Prove that for any orthonormal basis e_1, e_2 of a tangent plane to a surface, the mean curvature equals
$$\frac{1}{2}(\mathrm{II}(e_1, e_1) + \mathrm{II}(e_2, e_2)),$$
where $\mathrm{II}(\cdot, \cdot)$ is the second fundamental form of the surface.

   [Hint: Use Euler's theorem.]

3. Consider a surface for which both the Gaussian and the mean curvatures are identically equal to zero. Prove that any such surface is a subset of a plane.

   [Hint: A surface $r = r(u, v)$ is a subset of a plane if (a) its unit normal n is everywhere constant and (b) the radius vector r has constant projection on n, i.e., $\langle r, n \rangle$ is constant. First show that the second fundamental form is everywhere 0, then show (a) by differentiating $\langle r_u, n \rangle = 0$ and $\langle r_v, n \rangle = 0$.]