Homework 10 (submit by 15.06.2017)

1. Let

$$a(t) = \begin{cases} 2t & t \in [0, \frac{1}{2}] \\ 1 & t \in [\frac{1}{2}, 1]. \end{cases}$$

Find and classify the spectrum of the operator Ax(t) = a(t)x(t) acting on

- (a) C[0,1] (with supremum norm)
- (b) $L^2[0,1]$.
- 2. Let X be a Hilbert space. Let U be a unitary operator on X. Prove that
 - (a) ||U|| = 1
 - (b) $\sigma(U) \subseteq \{\lambda : |\lambda| = 1\}.$
- 3. Let X be a Hilbert space. Let P be an orthogonal projection. Find and classify the spectrum of P.
- 4. Let X be a Hilbert space. Let P be an orthogonal projection on a vector subspace L. Prove that P is compact if and only if $\dim(L) < \infty$.