

EXERCISES, Week 9 (submit by 12.12.2016)

1. Let γ be a (continuously differentiable) positively oriented boundary of a set $S \subset \mathbb{C}$ with area A . Compute the integral $\int_{\gamma} x dz$.
2. Compute the integral $\int_{\gamma} y dz$, where γ is
 - (a) the line segment between 0 and $2 + i$,
 - (b) the semicircle $|z| = 1$, $\text{Im}z \geq 0$ starting from the point $z = 1$.
3. Compute the integral $\int_{\gamma} \log z dz$, where you need to select a continuous branch of the logarithm based on the given value of the logarithm at one of the points on γ :
 - (a) γ is the unit circle $|z| = 1$ oriented counterclockwise and started at $z = 1$, and $\log 1 = 0$,
 - (b) γ is the unit circle $|z| = 1$ oriented counterclockwise and started at $z = i$, and $\log i = \frac{\pi i}{2}$.
4. Let γ be a positively oriented contour in \mathbb{C} . Compute $\int_{\gamma} \frac{dz}{z^2+9}$ if
 - (a) γ surrounds the point $3i$, but does not surround the point $-3i$,
 - (b) γ surrounds the point $-3i$, but does not surround the point $3i$,
 - (c) γ surrounds the points $3i$ and $-3i$,
 - (d) γ surrounds neither the point $3i$ nor $-3i$.
5. Compute the integral $\int_{\gamma} \frac{z dz}{z^4-1}$, where γ is a positively oriented circle $|z - a| = a$, and $a > 1$ is a real number.
6. Compute the integral $\frac{1}{2\pi i} \int_{\gamma} \frac{e^z dz}{z^2+a^2}$, where γ is a positively oriented contour surrounding the points ai and $-ai$. Here a is a positive real number.
7. Compute the integral $\frac{1}{2\pi i} \int_{\gamma} \frac{ze^z}{(z-a)^3} dz$, where γ is a positively oriented contour surrounding $a \in \mathbb{C}$.