EXERCISES, Week 6 (submit by 21.11.2016)

1. Compute the following surface integrals of scalar fields.
   (a) $\iint\limits_S (x+y+z)dS$, where S is the upper semisphere: $x^2+y^2+z^2=1$, $z\geq 0$.
   (b) $\iint\limits_S (x^2+y^2)\,dS$, where S is the full surface of the cone $\sqrt{x^2+y^2}\leq z\leq 1$.
   (c) $\iint\limits_S \frac{dS}{\sqrt{\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}}}$, where S is the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
   (d) $\iint\limits_S xyz\,dS$, where S is the surface of the cube $0\leq x\leq a$, $0\leq y\leq a$, $0\leq z\leq a$.
   (e) $\iint\limits_S z\,dS$, where S is part of the elliptic paraboloid $2z=x^2+y^2$ satisfying $z\leq 1$.
2. Compute the following surface integrals of vector fields.
   (a) $\iint\limits_S (x^2+y^2)\,dxdy$, where S is the bottom side of the disc $x^2+y^2\leq 4$, $z=0$.
   (b) $\iint\limits_S (2z-x)\,dydz+(x+2z)\,dzdx+3z\,dxdy$, where S is the upper side of the triangle $x+4y+z=4$, $x\geq 0$, $y\geq 0$, $z\geq 0$.
   (c) $\iint\limits_S x^2\,dydz$, where S is the outer side of the sphere $x^2+y^2+z^2=1$.
   (d) $\iint\limits_S (x-1)^3\,dydz$, where S is the inner side of the upper semisphere $x^2+y^2+z^2=2x$, $z\geq 0$.
   (e) $\iint\limits_S x\,dydz$, where S is the outer side of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.
   (f) $\iint\limits_S x^6\,dydz+y^4\,dzdx+z^2\,dxdy$, where S is the lower side of the elliptic paraboloid $z=x^2+y^2$, $z\leq 1$.