EXERCISES, Week 3 (submit by 01.11.2016)

1. By changing to polar coordinates, write $\iint_S f(x,y)dxdy$ as an iterated integral if $S = \{(x,y) \mid x^2 + y^2 \le 2ax, y \ge x\}$ $(a > 0)$.

2. Compute the integral $\iint_S f(x,y)dxdy$ using polar coordinates if

   (a) $f(x,y) = y^2 e^{x^2+y^2}$ and $S = \{(x,y) \mid x \ge 0,\ y \ge 0,\ x^2 + y^2 \le 1\}$,

   (b) $f(x,y) = x$ and $S = \{(x,y) \mid ax \le x^2 + y^2 \le 2ax,\ y \ge 0\}$ $(a > 0)$,

   (c) $f(x,y) = y$ and $S = \{(x,y) \mid 0 \le x \le (x^2 + y^2)^{\frac{3}{2}} \le 1,\ y \ge 0\}$.

3. Compute the integrals using the suggested change of variables

   (a) $\int_a^b dx \int_{\alpha x}^{\beta x} \frac{y}{x} dy$, where $0 < a < b$, $\alpha < \beta$. Change of variables: $u = x$, $v = \frac{y}{x}$.

   (b) $\int_0^1 dx \int_{-2-x}^{2-x} y dy$. Change of variables: $u = x$, $v = x + y$.

   (c) $\iint_S \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) dxdy$, where S is bounded by the curve $\begin{cases} x = a\sin t \\ y = b\cos t \end{cases}$, $0 \le t \le \frac{\pi}{2}$ and the axes $x = 0$, $y = 0$. Change of variables: $x = ar\sin t$, $y = br\cos t$.

   (d) $\iint_S xy\, dxdy$, where S is given by the inequalities $x \ge 0$, $y \ge 0$, $x^4 + y^4 \le a^4$ $(a > 0)$. Change of variables: $x = r\sqrt{\cos\varphi}$, $y = r\sqrt{\sin\varphi}$.

4. Compute the integral $\iint_S f(x,y)dxdy$ by making a suitable change of variables

   (a) $f(x,y) = xy$, $S = \{(x,y) \mid |x + 2y| \le 3,\ |x - y| \le 3\}$,

   (b) $f(x,y) = y^2$, $S = \{(x,y) \mid 1 \le xy \le 3,\ 0 < x \le y \le 2x\}$,

   (c) $f(x,y) = x$, $S = \{(x,y) \mid x \ge 0,\ y \ge 0,\ \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} \le 1\}$.

5. By changing to spherical coordinates, write $\iiint_S f(x,y,z)dxdydz$ as an iterated integral, if S is given by the inequalities $x^2 + y^2 + z^2 \le 2az$, $x^2 + y^2 \ge z^2$ $(a > 0)$.

6. Compute the integral $\iiint_S f(x,y,z)dxdydz$ using spherical or cylindrical coordinates

   (a) $f(x,y,z) = x^2 + y^2 - z^2$ and $S = \{(x,y,z) \mid 1 \le x^2 + y^2 + z^2 \le 4,\ x \ge 0,\ y \ge 0\}$,

   (b) $f(x,y,z) = x^2 + y^2$ and $S = \{(x,y,z) \mid \frac{x^2+y^2}{2} \le z \le 2\}$,

   (c) $f(x,y,z) = \sqrt{y^2 + z^2}$ and S is bounded by surfaces $y^2 + z^2 = 1$, $y + x = 1$, $y - x = 1$.