

**EXERCISES, Week 12** (submit by 16.01.2017)

1. Identify the type of the following PDEs.

(a)  $\sin^2 y u_{xx} - e^{2x} u_{yy} + u_x = 0$

(b)  $(x - y) u_{xx} + (xy - y^2 - x + y) u_{xy} = 0$

2. For each of the following PDEs, find a change of variables that brings it to its canonical form.

(a)  $y u_{xx} - x u_{yy} = 0$

(b)  $y^2 u_{xx} - e^{2x} u_{yy} + u_x = 0$

3. Write the following PDEs in their canonical forms.

(a)  $e^{2x} u_{xx} + 2e^{x+y} u_{xy} + e^{2y} u_{yy} + u_y = 0$

(b)  $u_{xx} + xy u_{yy} = 0$

4. Simplify the following PDE by changing the unknown function to  $v(x, y)$ , where  $u(x, y) = e^{\lambda x + \mu y} v(x, y)$ , and choosing the parameters  $\lambda, \mu$  suitably.

$$u_{xx} + u_{yy} + \alpha u_x + \beta u_y + \gamma u = 0.$$