EXERCISES, Week 11 (submit by 09.01.2017)

1. Use the residue theorem to compute the following integrals.

(a) $\oint\limits_{|z-2|=\frac{1}{2}} \dfrac{zdz}{(z-1)(z-2)^2}$ (b) $\oint\limits_{|z|=1} \dfrac{z^3dz}{2z^4+1}$ (c) $\dfrac{1}{2\pi i} \oint\limits_{|z|=2} \sin\dfrac{1}{z}\, dz$

(d) $\dfrac{1}{2\pi i} \oint\limits_{|z|=2} \sin^2\dfrac{1}{z}\, dz$ (e) $\oint\limits_{|z|=5} \dfrac{zdz}{\sin z(1-\cos z)}$

2. Compute the following integrals.

(a) $\displaystyle\int_0^{2\pi} \dfrac{d\varphi}{a+\cos\varphi}\ (a>1)$ (b) $\displaystyle\int_0^\pi \tan(x+ia)dx\ (a\in\mathbb{R}, a\neq 0)$

(c) $\displaystyle\int_0^\infty \dfrac{x^2dx}{(x^2+a^2)^2}\ (a>0)$ (d) $\displaystyle\int_{-\infty}^\infty \dfrac{dx}{(x^2+a^2)(x^2+b^2)}\ (a>0, b>0)$

(e) $\displaystyle\int_{-\infty}^\infty \dfrac{x\cos xdx}{x^2-2x+10}$ (f) $\displaystyle\int_{-\infty}^\infty \dfrac{x\sin xdx}{x^2-2x+10}$ (g) $\displaystyle\int_0^\infty \dfrac{x\sin axdx}{x^2+b^2}\ (a,b>0)$