

EXERCISES, Week 10 (submit by 19.12.2016)

1. Expand the function $\frac{z^2}{(z+1)^2}$ in the power series

$$(a) \sum_{n=0}^{\infty} a_n z^n \quad (b) \sum_{n=0}^{\infty} b_n (z-1)^n$$

and find the radii of convergence of the series (a) and (b).

2. Find all zeros and their orders for the following functions:

$$(a) 1 - \cos z \quad (b) z \sin z \quad (c) z^2(e^{z^2} - 1).$$

3. Does there exist a function f holomorphic at $z = 0$ and such that $f(\frac{1}{n})$, $n \geq 1$, equals

$$(a) 0, 1, 0, 1, 0, 1, 0, 1, \dots$$

$$(b) 0, \frac{1}{2}, 0, \frac{1}{4}, 0, \frac{1}{6}, 0, \frac{1}{8}, \dots$$

$$(c) \frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6}, \frac{1}{6}, \frac{1}{8}, \frac{1}{8}, \dots$$

$$(d) \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$$

Justify your answers.

4. Find all isolated singularities for the following functions and determine their types:

$$(a) \frac{1}{z(1+z^2)} \quad (b) ze^{\frac{1}{z}} \quad (c) \frac{1}{\sin z} \quad (d) \cos \frac{1}{z} \quad (e) \sin \frac{1}{z} + \frac{1}{z}.$$

5. Find the Laurent series for the following functions:

$$(a) \frac{1}{z+2} \text{ in}$$

$$(i) 0 < |z| < 2 \quad (ii) 2 < |z| < \infty$$

$$(b) \frac{1}{z(1-z)} \text{ in}$$

$$(i) 0 < |z| < 1 \quad (ii) 0 < |z-1| < 1$$

$$(c) \frac{1}{(z-a)(z-b)} \text{ (here } a, b \in \mathbb{C} \text{ and } 0 < |a| < |b| \text{) in}$$

$$(i) 0 < |z| < |a| \quad (ii) 0 < |z-a| < |b-a| \quad (iii) |a| < |z| < |b|$$

$$(d) z^2 \sin \frac{1}{z-1} \text{ in } 0 < |z-1| < \infty.$$