**EXERCISES, Week 8** (submit by 06.12.2017)

1. Let $S$ be the smooth boundary of a solid object in $\mathbb{R}^3$ and $\nu$ its outward unit normal. Let $\ell$ be a fixed vector of $\mathbb{R}^3$. Prove that $\iint\limits_S \cos(\widehat{\ell,\nu})\,dS = 0$.

   (Here $\widehat{\ell,\nu}$ denotes the angle between $\ell$ and $\nu$.)

2. Use the Stokes theorem to compute the following line integrals.

   (a) $\int\limits_\gamma (x+z)dx + (x-y)dy + xdz$, where $\gamma$ is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $z = 1$ oriented counter-clockwise viewed from the point $(0,0,0)$.

   (b) $\int\limits_\gamma y^2dx + z^2dy + x^2dz$, where $\gamma$ is the triangle with vertices $(a,0,0)$, $(0,a,0)$, $(0,0,a)$ oriented clockwise viewed from the point $(0,0,0)$.

   (c) $\int\limits_\gamma ydx + zdy + xdz$, where $\gamma$ is the circle $x^2+y^2+z^2 = R^2$, $x+y+z = 0$ oriented counter-clockwise viewed from the point $(0,0,1)$.

   (d) $\int\limits_\gamma z^2dy + x^2dz$, where $\gamma$ is the curve $y^2+z^2 = 9$, $4x+3z = 5$ oriented clockwise viewed from the point $(0,0,0)$.

3. (a) Let $u$ and $v$ be scalar fields on $\mathbb{R}^3$. Let $F = u\nabla v$. Prove that $\nabla \times F$ is orthogonal to $F$.

   (b) Let $F$ be an arbitrary vector field on $\mathbb{R}^3$. Is $\nabla \times F$ orthogonal to $F$?