EXERCISES, Week 6-7 (submit by 29.11.2017)

1. Compute the following surface integrals of scalar fields.

   (a) $\iint\limits_S xyz\, dS$, where S is the surface of the cube $0 \le x \le a$, $0 \le y \le a$, $0 \le z \le a$.

   (b) $\iint\limits_S (x^2 + y^2)\, dS$, where S is the full surface of the cone $\sqrt{x^2 + y^2} \le z \le 1$.

   (c) $\iint\limits_S \frac{dS}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}}$, where S is the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

2. Compute the following surface integrals of vector fields.

   (a) $\iint\limits_S (x^2 + y^2)\, dxdy$, where S is the bottom side of the disc $x^2 + y^2 \le 4$, $z = 0$.

   (b) $\iint\limits_S (2z - x)\, dydz + (x + 2z)\, dzdx + 3z\, dxdy$, where S is the upper side of the triangle $x + 4y + z = 4$, $x \ge 0$, $y \ge 0$, $z \ge 0$.

   (c) $\iint\limits_S x^6\, dydz + y^4\, dzdx + z^2\, dxdy$, where S is the lower side of the elliptic paraboloid $z = x^2 + y^2$, $z \le 1$.

3. Use the Gauss-Ostrogradsky theorem to compute the following surface integrals.

   (a) $\iint\limits_S (1 + 2x)dydz + (2x + 3y)dzdx + (3y + 4z)dxdy$, where S is the outer side of the boundary of the pyramid $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \le 1$, $x \ge 0$, $y \ge 0$, $z \ge 0$.
   (Here a, b, c are arbitrary positive numbers.)

   (b) $\iint\limits_S x^2dydz + y^2dzdx + z^2dxdy$, where S is the lower side of the semisphere $x^2 + y^2 + z^2 = R^2$, $z \ge 0$.