

EXERCISES, Week 5 (submit by 15.11.2017)

1. Use Green's formula to compute $I = \int_{\gamma} P dx + Q dy$.
 - (a) $I = \int_{\gamma} (3x^2 - y) dx + (1 - 2x) dy$, where γ is the positively oriented boundary of the triangle with vertices at $(0, 0)$, $(1, 0)$, $(0, 1)$.
 - (b) $I = \int_{\gamma} \frac{x dy + y dx}{x^2 + y^2}$, where γ is the circle $(x - 1)^2 + (y - 1)^2 = 1$ oriented counter-clockwise.
 - (c) $I = \int_{\gamma} (e^x \sin y - y) dx + (e^x \cos y - 1) dy$, where γ is the positively oriented boundary of the set $\{(x, y) \mid x^2 + y^2 < ax, y > 0\}$. (Here $a > 0$.)
2. Let γ be a curve connecting A to B . Compute $I = \int_{\gamma} P dx + Q dy$ by identifying a scalar potential for (P, Q) .
 - (a) $I = \int_{\gamma} x dy + y dx$, $A = (-1, 3)$, $B = (2, 2)$.
 - (b) $I = \int_{\gamma} x dx + y dy$, $A = (-1, 0)$, $B = (-3, 4)$.
 - (c) $I = \int_{\gamma} \frac{x dx + y dy + z dz}{\sqrt{x^2 + y^2 + z^2}}$, A lies on the sphere $x^2 + y^2 + z^2 = r^2$, B lies on the sphere $x^2 + y^2 + z^2 = R^2$.
3. Find the area bounded by the curve
 - (a) $x = 12 \sin^3 t$, $y = 3 \cos^3 t$, $t \in [0, 2\pi]$.
 - (b) $x = \frac{3t}{1+t^3}$, $y = \frac{3t^2}{1+t^3}$, $t > 0$.