EXERCISES, Week 3 (submit by 01.11.2017)

1. Write $\iiint_S f(x,y,z)dxdydz$ as an iterated integral, if S is given by the inequalities $x \geq 0$, $z \geq 0$, $x^2+y^2 \leq a^2$, $y^2+z^2 \leq a^2$.

2. Compute the multiple integrals

   (a) $\iiint_S (xy)^2 dxdydz$, where S is given by the inequalities $0 \leq x \leq y \leq z \leq 1$.

   (b) $\iiint_S (x+2y+3z)dxdydz$, where S is the prism bounded by the planes $y=0$, $z=0$, $z=2$, $x+y=2$, $2x-y+2=0$.

3. Compute the integral $\iint_S f(x,y)dxdy$ using polar coordinates if

   (a) $f(x,y)=x$ and $S=\{(x,y) \mid ax \leq x^2+y^2 \leq 2ax,\ y \geq 0\}$ $(a>0)$,

   (b) $f(x,y)=y$ and $S=\{(x,y) \mid 0 \leq x \leq (x^2+y^2)^{\frac{3}{2}} \leq 1,\ y \geq 0\}$.

4. Compute the integrals using the suggested change of variables

   (a) $\iint_S \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right) dxdy$, where S is bounded by the curve $\begin{cases} x = a\sin t \\ y = b\cos t \end{cases}$, $0 \leq t \leq \frac{\pi}{2}$ and the axes $x=0$, $y=0$. Change of variables: $x = ar\sin t$, $y = br\cos t$.

   (b) $\iint_S xy\, dxdy$, where S is given by the inequalities $x \geq 0$, $y \geq 0$, $x^4+y^4 \leq a^4$ $(a>0)$. Change of variables: $x = r\sqrt{\cos\varphi}$, $y = r\sqrt{\sin\varphi}$.

5. Compute the integral $\iint_S f(x,y)dxdy$ by making a suitable change of variables

   (a) $f(x,y)=xy$, $S=\{(x,y) \mid |x+2y| \leq 3,\ |x-y| \leq 3\}$,

   (b) $f(x,y)=x$, $S=\{(x,y) \mid x \geq 0,\ y \geq 0,\ \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} \leq 1\}$.