EXERCISES, Week 2 (submit by 25.10.2017)

1. Let g be an integrable function on $[a,b]$ and h an integrable function on $[c,d]$ so that $\int_a^b g(z)dz = \int_c^d h(z)dz = 1$. Let $S = [a,b]\times[c,d]$ be a rectangle in $\mathbb{R}^2$. Use the definition of Riemann integral to compute $\int_S f(x)dx$ if

$$\text{(a)}\quad f(x) = g(x_1) + h(x_2), \qquad \text{(b)}\quad f(x) = g(x_1)\,h(x_2).$$

2. Compute the iterated integral

$$\int_{-\pi}^{\pi} dx \int_{\sin x}^{\cos x} dy \int_{y+x}^{y-x} (x+y+z)dz\,.$$

3. Write the integral $\iint_S f(x,y)\,dxdy$ as an iterated integral, if S is
   - (a) the triangle bounded by the lines $x=0$, $y=0$, $ax+by=c$,
   - (b) bounded by the curves $x=0$, $x=1$, $x=y^2$, $y=e^x$.

4. Change the order of integration.

$$\text{(a)}\ \int_0^1 dy \int_0^{y^2} f(x,y)dx \qquad \text{(b)}\ \int_1^2 dx \int_{\ln x}^{3x} f(x,y)dy\,.$$

5. Compute the multiple integrals
   - (a) $\iint_S x^2y^2dxdy$, where S is bounded by the curves $x=1$, $x=y^2$.
   - (b) $\iint_S (x+y)dxdy$, where S is given by the inequalities $x^2+y^2\le 1$, $y\ge x$.
   - (c) $\iint_S |xy|dxdy$, where S is given by the inequalities $a^2\le x^2+y^2\le b^2$ $(0<a<b)$.