EXERCISES, Week 12 (submit by 17.01.2018)

1. Using only the definition of isolated singularities, determine the type of all isolated singularities of the following functions:

$$\text{(a) } \frac{1}{z(1+z^2)} \qquad \text{(b) } \frac{z}{\sin z} \qquad \text{(c) } \cos\frac{1}{z}$$

2. Find the Laurent series for the following functions:

   (a) $\frac{1}{z+2}$ in

   $$\text{(i) } 0 < |z| < 2 \qquad \text{(ii) } 2 < |z| < \infty$$

   (b) $\frac{1}{z(1-z)}$ in

   $$\text{(i) } 0 < |z| < 1 \qquad \text{(ii) } 0 < |z-1| < 1$$

   (c) $z^2 \sin\frac{1}{z-1}$ in $0 < |z-1| < \infty$.

3. Use the residue theorem to compute the following integrals.

$$\text{(a) } \oint_{|z-2|=\frac{1}{2}} \frac{zdz}{(z-1)(z-2)^2} \qquad \text{(b) } \oint_{|z|=1} \frac{z^3dz}{2z^4+1}$$

$$\text{(c) } \frac{1}{2\pi i} \oint_{|z|=2} \sin\frac{1}{z}\,dz \qquad \text{(d) } \oint_{|z|=5} \frac{zdz}{\sin z(1-\cos z)}$$