

**EXERCISES, Week 11** (submit by 10.01.2018)

1. Use Cauchy's integral formula for derivatives to compute the integral

$$\frac{1}{2\pi i} \int_{\gamma} \frac{ze^z}{(z-a)^3} dz,$$

where  $\gamma$  is a positively oriented contour surrounding  $a \in \mathbb{C}$ .

2. Expand the function  $\frac{z^2}{(z+1)^2}$  in the power series

$$(a) \sum_{n=0}^{\infty} a_n z^n \quad (b) \sum_{n=0}^{\infty} b_n (z-1)^n$$

and find the radii of convergence of the series (a) and (b).

3. Find all zeros and their orders for the following functions:

$$(a) 1 - \cos z \quad (b) z \sin z \quad (c) z^2(e^{z^2} - 1).$$

4. Does there exist a function  $f$  holomorphic at  $z = 0$  and such that  $f(\frac{1}{n})$ ,  $n \geq 1$ , equals

$$(a) 0, 1, 0, 1, 0, 1, 0, 1, \dots$$

$$(b) 0, \frac{1}{2}, 0, \frac{1}{4}, 0, \frac{1}{6}, 0, \frac{1}{8}, \dots$$

$$(c) \frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{6}, \frac{1}{6}, \frac{1}{8}, \frac{1}{8}, \dots$$

$$(d) \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$$

Justify your answers.