

EXERCISES, Week 10 (submit by 03.01.2018)

1. Let $f(z) = z^2$.
 - (a) Determine the angle of rotation of the complex plane by f at the point $z = 1+i$.
 - (b) Which part of the complex plane is stretched and which is contracted by f ?
2. Let γ be a (continuously differentiable) positively oriented boundary of a set $S \subset \mathbb{C}$ with area A . Compute the integral $\int_{\gamma} x dz$.
3. Compute the integral $\int_{\gamma} y dz$, where γ is
 - (a) the line segment between 0 and $2+i$,
 - (b) the semicircle $|z| = 1$, $\text{Im} z \geq 0$ starting from the point $z = 1$.
4. Compute the integral $\int_{\gamma} \log z dz$, where you need to select a continuous branch of the logarithm based on the given value of the logarithm at one of the points on γ :
 - (a) γ is the unit circle $|z| = 1$ oriented counterclockwise and started at $z = 1$, and $\log 1 = 0$,
 - (b) γ is the unit circle $|z| = 1$ oriented counterclockwise and started at $z = i$, and $\log i = \frac{\pi i}{2}$.
5. Let γ be a positively oriented contour in \mathbb{C} . Use Cauchy's integral formula to compute $\int_{\gamma} \frac{dz}{z^2+9}$ if
 - (a) γ surrounds the point $3i$, but does not surround the point $-3i$,
 - (b) γ surrounds the point $-3i$, but does not surround the point $3i$,
 - (c) γ surrounds the points $3i$ and $-3i$,
 - (d) γ surrounds neither the point $3i$ nor $-3i$.
6. Use Cauchy's integral formula to compute the integral $\int_{\gamma} \frac{z dz}{z^4-1}$, where γ is a positively oriented circle $|z-a| = a$, and $a > 1$ is a real number.