# 52nd Seminar Sophus Lie
# Abstracts

1. Johannes Huebschmann (Lille):

   *Lie-Rinehart algebras and Atiyah class*

   For a commutative ring $R$ and a commutative $R$-algebra $A$, an $(R, A)$-Lie algebra is an $R$-Lie algebra $L$, together with a structure of mutual interaction between $A$ and $L$ that arises by abstracting from the special case $(A, L) = (C^\infty(M), \mathrm{Vect}(M))$ where $C^\infty(M)$ is the algebra of smooth functions and $\mathrm{Vect}(M)$ the Lie algebra of smooth vector fields on a smooth manifold $M$. When $L$ is an $(R, A)$-Lie algebra, the pair $(A, L)$ is said to be a Lie-Rinehart algebra.

   Let $H \to L$ be an injection of $(R, A)$-Lie algebras. A suitable notion of *Atiyah class of the injection* $H \subseteq L$ generalizes various notions of Atiyah class in the literature. Atiyah introduced such a class as the obstruction for a holomorphic connection on a holomorphic principal bundle to exist and, somewhat later, Molino and, independently, Kamber-Tondeur, introduced such a class as the obstruction for a projectable transverse connection to exist on a foliation.

   Suppose that $L$ and $L/H$ are projective as $A$-modules. The bracket operation of $L$ induces, on $L/H$, an $(A, H)$-module structure and hence, on the symmetric $A$-coalgebra $\mathrm{S}^{\mathrm{c}}_A[L/H]$ on $L/H$, an $(A, H)$-module structure, and that structure is compatible with the $A$-coalgebra structure. On the other hand, the operation of multiplication in the universal algebra $\mathrm{U}_A[L]$ of $(A, L)$ (the algebra of differential operators on $M$ when $(A, L) = (C^\infty(M), \mathrm{Vect}(M))$ for a smooth manifold $M$) induces, on $\mathrm{S}^{\mathrm{c}}_A[L/H]$, an $(A, H)$-module structure that is compatible with the $A$-coalgebra structure and is, furthermore, a perturbation of the former structure in an obvious sense. The "first term of the perturbed structure" recovers the Atiyah class, and the main result says that the perturbed structure is isomorphic to the standard structure if and only if the Atiyah class of the injection $H \to L$ vanishes. The proof of the fact that the perturbed structure reduces to the standard structure if the Atiyah class of the injection $H \to L$ vanishes—picturally, that the first term of the perturbed structure controls the entire perturbed structure—is rather hard and necessitates pushing further the existing structure theory of Lie-Rinehart algebras, involves a suitable notion of a coalgebra over a certain bialgebra, and leads among others to Lie-Rinehart generalizations of the bialgebra Cartier-Kostant-Milnor-Moore structure theorem saying that the assignment to a Lie algebra of its universal algebra establishes an equivalence of categories between Lie algebras and certain bialgebras (Hopf algebras). An equivalent $L_\infty$-variant of the theory abstracts from and generalizes an approach in the literature to the Rosansky-Witten invariants for 3-manifolds in terms of certain $L_\infty$-algebras.

   The purpose of the talk is to give an overview of these ideas.

2. Ines Kath (Greifswald):

   *Compact quotients of Cahen-Wallach spaces*

   Indecomposable symmetric Lorentzian manifolds of non-constant curvature are called Cahen-Wallach spaces. Their isometry classes are described by continuous families of real parameters. We derive necessary and sufficient conditions for the existence of compact quotients of Cahen-Wallach spaces in terms of these parameters (joint work with Martin Olbrich).

3. Jasmin Matz (Jersualem):

   The analytic torsion of a compact Riemannian manifold is a certain invariant which is defined via the Laplace-Beltrami operator. By the Cheeger-Müller theorem it equals the Reidemeister torsion of that manifold and can therefore often be used to study properties of certain arithmetic lattices as, for example, in the recent work of Bergeron and Venkatesh. It is therefore of interest to extend the notion of analytic torsion to non-compact locally symmetric spaces. For finite volume hyperbolic manifolds this was achieved by various people over the last years, but no case of rank $\geq 2$ was considered. In my talk I want to explain how one can define the analytic torsion for congruence quotients of the symmetric space $X := \mathrm{SL}_n(\mathbf{R})/\mathrm{SO}(n)$ with coefficients in strongly acyclic bundles. Further, I will discuss the behavior of the analytic torsion in the limit $k \to \infty$ for the spaces $\Gamma_k \backslash X$ with $\Gamma_k$ varying over a sequence of congruence subgroups with $\mathrm{vol}(\Gamma_k \backslash X) \longrightarrow \infty$. (Joint work with W. Müller.)

4. Anke Pohl (Jena):

   *Bounds for elementary spherical functions*

   We discuss bounds for elementary spherical functions that are uniform as spectral parameters approach the walls of Weyl chambers as well as the argument approaches the identity. This is joint work with Valentin Blomer.

5. Felix Pogorzelski (Leipzig):

   *Quasicrystals beyond abelian groups*

   The investigation of aperiodic point sets goes back to work of Yves Meyer who pursued analysis on harmonious sets in Euclidean space. Dan Shechtman's discovery of physical quasicrystals (1982) via laser experiments (diffraction) triggered a boom of the mathematical analysis of the arising phenomena. In recent work with Michael Björklund and Tobias Hartnick, we have developed a spherical diffraction theory for cut-and-project sets in general lcsc groups, thus advancing into the world of quite general symmetric spaces. This talk is devoted to a brief outline of the theory.

   Joint work with Michael Björklund and Tobias Hartnick.

6. Henrik Schlichtkrull (Copenhagen):

   *Lie groups, symmetric spaces and beyond*

   When we encounter a Lie group it is usually as a group $G$ which acts on some other object $X$ (in the language of Sophus Lie, a *continuous transformation group*). If the group acts transitively then $X$ can be identified with a homogeneous space $G/H$. For example, the classical situation of isometries of symmetric spaces was studied by Killing and Cartan around 1900. Today the Lie group $G$ serves mainly as an important tool for doing analysis on $X$. Harmonic analysis on symmetric spaces was fully developed in the second half of the 20th century by Harish-Chandra and others. Recent investigations (together with F.Knop and B.Krtz) suggest a wider class of homogeneous spaces of Lie groups, which is accessible for doing harmonic analysis.