UNIVERSITÄT LEIPZIG

Problem sheet 13

Solutions has to be uploaded into Moodle:
https://moodle2.uni-leipzig.de/mod/assign/view.php?id=1117957
until 22:00, July 15.

1. [**5 points**] Let $T : l^2 \to l^2$ be defined by

$$Tx = (\alpha_k \xi_k)_{k \geq 1}, \quad x = (\xi_k)_{k \geq 1},$$

   where $\{\alpha_n\}_{n \geq 1}$ is a bounded sequence in $\mathbb{C}$. Prove that T is compact if and only if $\alpha_n \to 0$.

2. Let $T : \mathcal{D}(T) \to l^2$ be defined by

$$Tx = (k\xi_k)_{k \geq 1}, \quad x = (\xi_k)_{k \geq 1},$$

   where $\mathcal{D}(T) \subset l^2$ consists of all $x = (\xi_k)_{k \geq 1}$ with only finitely many nonzero terms ξ_k.

   (a) [**5 points**] Show that T is unbounded and not closed. Find the adjoint operator T^* of T.

   (b) [**4 bonus points**] Show that T is closable and find its closure $\bar{T}$. Find the adjoint operator $\bar{T}^*$ of $\bar{T}$.