Problem sheet 6

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Solutions will be collected during the lecture on Tuesday December 3.

1. **[3+4+4 points]** Evaluate the following surface integrals
   (a) $\iint\limits_S (2z-x)dydz + (x+2z)dzdx + 3zdxdy$, where S is the upper side (oriented up) of the triangle $x+4y+z=4$, $x \geq 0$, $y \geq 0$, $z \geq 0$.
   (b) $\iint\limits_S \left(\frac{dydz}{x} + \frac{dzdx}{y} + \frac{dxdy}{z}\right)$, where S is the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ oriented outward;
   (c) $\iint\limits_S (y-z)dydz + (z-x)dzdx + (x-y)dxdy$, where S is the surface $x^2+y^2=z^2$ $(0 \leq z \leq h)$ oriented outward.

2. **[3+3+5 points]** Using the Gauss-Ostrogradskii divergence theorem evaluate the following integrals
   (a) $\iint\limits_S x^2dydz + y^2dzdx + z^2dxdy$, where S is the boundary of the cube $0 \leq x \leq a$, $0 \leq y \leq a$, $0 \leq z \leq a$ oriented outward.
   (b) $\iint\limits_S xy^2dydz + yz^2dzdx + zx^2dxdy$, where S is the sphere $x^2+y^2+z^2=R^2$ oriented outward.
   (c) $\iint\limits_S x^2dydz + y^2dzdx + z^2dxdy$, where S is the part of the cone $x^2+y^2=z^2$ $(0 \leq z \leq h)$ oriented outward.

   (*Hint:* Add the part of plane $z=h$, $x^2+y^2 \leq h^2$.)