Problem sheet 4

Tutorials by Mohammad Hashemi <hashemi@math.uni-leipzig.de>.
Solutions will be collected during the lecture on Monday November 18.

1. **[2 points]** Compute the line integral $\int_\gamma xyds$, where γ is the part of the circle $x^2 + y^2 = 1$ located in the positive quadrant $\{(x, y) : \ x \geq 0, y \geq 0\}$.

2. **[3 points]** Compute the line integral $\int_\gamma zds$, where γ is the helix in $\mathbb{R}^3$, $\{(x, y, z) : \ x = t\cos t, \ y = t\sin t, \ z = t, \ 0 \leq t \leq 2\pi\}$.

3. **[2 points]** Compute $\int_\gamma 2xydx + x^2dy$, where γ is the oriented curve $\left\{(x, y) : \ y = \frac{x^2}{4}, \ 0 \leq x \leq 2\right\}$ with the orientation from $x = 0$ to $x = 2$.

4. **[3 points]** Compute $\int_\gamma (y + z)dx + (z + x)dy + (x + y)dz$, where γ is the oriented curve $\{(x, y, z) : x = \sin^2 t, \ y = 2\sin t\cos t, \ z = \cos^2 t, \ 0 \leq t \leq \pi\}$ with the orientation from $t = 0$ to $t = \pi$.

5. **[3 points]** Using Green's theorem, evaluate $\oint\limits_\gamma (x + y)dx - (x - y)dy$, where γ is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ oriented counter clockwise.

6. **[5 points]** Evaluate $\oint\limits_\gamma \frac{xdy - ydx}{x^2+y^2}$, where γ is a simple closed curve that does not pass through the origin and is oriented counter clockwise.

   (*Hint:* Let S be the domain surrounded by γ. For the case $(0, 0) \notin S$ just use the Green's theorem for S. If $(0, 0) \in S$ then apply the Green's theorem for the domain $S \setminus B_\varepsilon(0, 0)$ and then make $\varepsilon \to 0$)

7. **[3 points]** Using Green's theorem, compute area of the domain bounded by the astroid $x = a\cos^3 t, \ y = b\sin^3 t$ $(0 \leq t \leq 2\pi)$.