Problem sheet 9

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Solutions will be collected during the lecture on Wednesday January 9.

1. **[1x3 points]** Find points of local extrema of the following functions:
   a) $f(x) = x^2 e^x$, $x \in \mathbb{R}$; b) $f(x) = x + \frac{1}{x}$, $x > 0$; c) $f(x) = |x| e^{-x^2}$, $x \in \mathbb{R}$.

2. **[1x3 points]** Identify intervals on which the following functions are convex or concave:
   a) $f(x) = \frac{1}{x}$, $x > 0$; b) $f(x) = \arctan x$, $x \in \mathbb{R}$.

3. **[3 points]** *(Young's inequality)* Let $p > 1$, $q > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$.
   Prove that $xy \leq \frac{x^p}{p} + \frac{y^q}{q}$ for all $x, y \in (0, +\infty)$.
   (*Hint:* Consider the function $f(x) = -\ln x$, $x > 0$, and use its convexity on $(0, +\infty)$)

4. **[2x9 points]** Compute the following indefinite integrals:
   a) $\int \frac{dx}{x \ln x}$, $x > 0$; b) $\int \frac{(2x+1)dx}{\sqrt[3]{1+x+x^2}}$, $x \in \mathbb{R}$; c) $\int \frac{\sqrt{\tan x}}{\cos^2 x} dx$, $x \in \left(0, \frac{\pi}{2}\right)$;
   d) $\int \frac{dx}{\cos x}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; e) $\int \cos^2 x \sin^3 x dx$, $x \in \mathbb{R}$; f) $\int \frac{dx}{x^2+x+1}$, $x \in \mathbb{R}$;
   g) $\int x^2 \sin x dx$, $x \in \mathbb{R}$; h) $\int (\ln x)^2 dx$, $x > 0$; i) $\int e^{2x} \cos x dx$, $x \in \mathbb{R}$.