Problem sheet 13

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Solutions will be collected during the lecture on Monday February 4.

1. **[2 points]** For a complex number α show that the coefficients of the polynomial

$$p(z) = (z - \alpha)(z - \overline{\alpha})$$

   are real numbers.

2. **[3 points]** Let $p(z)$ be a polynomial with real coefficients and let α be a complex number. Prove that $p(\alpha) = 0$ if and only if $p(\overline{\alpha}) = 0$.

3. **[3 points]** Prove that any polynomial $p(z)$ with real coefficients can be decomposed into a product of polynomials of the form $az^2 + bz + c$, where $a, b, c \in \mathbb{R}$.

   (*Hint:* Use the fundamental theorem of algebra and exercises 1., 2.)

4. **[6x2 points]** For each of the following sets, either show that the set is a vector space over $\mathbb{F}$ or explain why it is not a vector space.

   a) The set $\mathbb{R}$ of real numbers under the usual operations of addition and multiplication, $\mathbb{F} = \mathbb{R}$.
   b) The set $\mathbb{R}$ of real numbers under the usual operations of addition and multiplication, $\mathbb{F} = \mathbb{C}$.
   c) The set $\{f \in \mathrm{C}[0,1] : f(0) = 2\}$ under the usual operations of addition and multiplication of functions, $\mathbb{F} = \mathbb{R}$.
   d) The set $\{f \in \mathrm{C}[0,1] : f(0) = f(1) = 0\}$ under the usual operations of addition and multiplication of functions, $\mathbb{F} = \mathbb{R}$.
   e) The set $\{(x, y, z) \in \mathbb{R}^3 : x - 2y + z = 0\}$ under the usual operations of addition and multiplication on $\mathbb{R}^3$, $\mathbb{F} = \mathbb{R}$.
   f) The set $\{(x, y, z) \in \mathbb{C}^3 : 2x + z + i = 0\}$ under the usual operations of addition and multiplication on $\mathbb{C}^3$, $\mathbb{F} = \mathbb{C}$.