Problem sheet 1

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Solutions will be collected during the lecture on Monday October 29.

1. **[1+1 points]** List elements of the following sets:
   a) $\{n \in \mathbb{N} : \ (n-4)^2 < 5^2\}$; b) $\{n \in \mathbb{N} : \ n^3 > 4n\}$.

2. **[2+2+2 points]** Check the following relations:
   a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$; b) $(A \cup B)^c = A^c \cap B^c$; c) $\left(\bigcap\limits_{t \in T} A_t\right)^c = \bigcup\limits_{t \in T} A_t^c$

3. **[2+2+2 points]** Prove that
   a) $\sqrt{6} \notin \mathbb{Q}$; b)$\sqrt{2} + \sqrt{3} \notin \mathbb{Q}$; c) for each $n \in \mathbb{N}$ either $\sqrt{n} \in \mathbb{N}$ or $\sqrt{n} \notin \mathbb{Q}$.

4. **[3+3 points]** Using mathematical induction prove that:
   a) $1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$ for each $n \in \mathbb{N}$;
   b) $11^n - 4^n$ is divisible by 7 for each $n \in \mathbb{N}$.

5. **[2+2+3 points]** Prove that a) $\sup A = -\inf(-A)$, where A is a subset of $\mathbb{R}$ bounded from above and $-A := \{-a : \ a \in A\}$;
   b) Let A and B be subsets of $\mathbb{R}$ bounded from above. Show that $\sup(A \cup B) = \max\{\sup A, \sup B\}$;
   c) Let $A = \{0, \alpha_1\alpha_2 \ldots \alpha_n \ldots : \ \forall n \in \mathbb{N} \ \ \alpha_n \in \{1, 2, 3, 4, 5, 6, 7, 8\}\}$. Find $\inf A$ and $\sup A$.