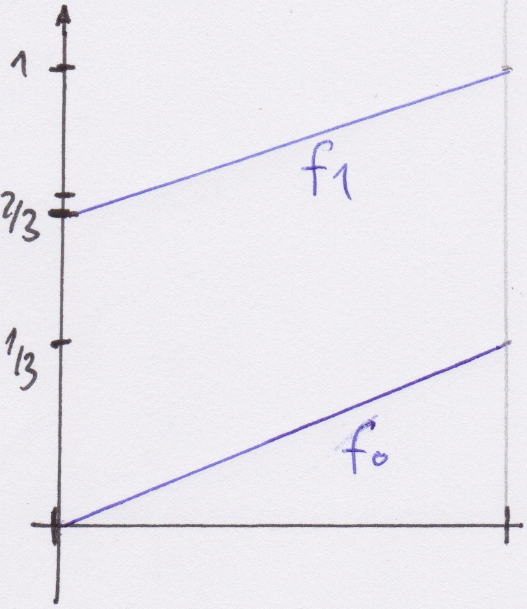
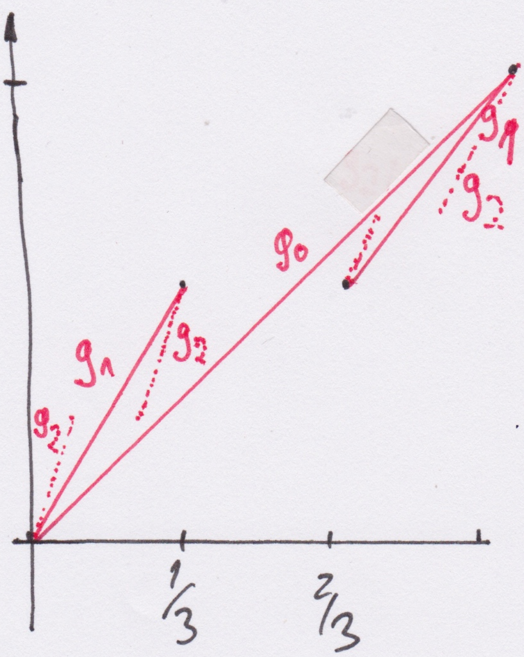
(I)

Beweis der Eigenschaften von g_n, $n \geq 0$, Induktionsschritt





a) wenn $x \in f_i(C_n) \Rightarrow g_{n+1}(x) = \left[g_n(f_i^{-1}(x)) + i\right]/2$

also g_{n+1} nichtfallend auf $f_0(C_n)$ & $f_1(C_n)$, da

$g_n \nearrow$ & f_i^{-1} genauso wie $f_i \nearrow$

falls $x \in f_i(C_n)$ & $y \in f_{1-i}(C_n)$ & $x \leq y \Rightarrow i = 0$, also

$g_{n+1}(x) = g_n(f_0^{-1}(x))/2 \leq \frac{1}{2}$ & $\frac{1}{2} \leq \left[g_n(f_1^{-1}(y)) + 1\right]/2 = g_{n+1}(y)$

denn $f_0(C_n) \subset [0, \frac{1}{3}]$ & $f_1(C_n) \subset [\frac{2}{3}, 1]$, also $g_{n+1} \nearrow$ auf C_{n+1}

b) $g_{n+1}(C_{n+1}) = g_{n+1}(f_0(C_n)) \cup g_{n+1}(f_1(C_n))$

$$= \frac{g_n(C_n) + 0}{2} \cup \frac{g_n(C_n) + 1}{2} = \frac{[0,1]}{2} \cup \frac{[0,1]+1}{2} = [0,1]$$

c) die interessanteste Behauptung

$$(*)\left\{\begin{array}{l} \bullet\ x, y \in f_i(C_n) \Rightarrow |g_{n+1}(x) - g_{n+1}(y)| = \frac{1}{2}\left|g_n(f_i^{-1}(x)) - g_n(f_i^{-1}(y))\right| \\ \quad \leq \frac{1}{2}\left|f_i^{-1}(x) - f_i^{-1}(y)\right|^d \leq \frac{1}{2}\left|3(x-y)\right|^d = \frac{3^d}{2}|x-y|^d = |x-y|^d \end{array}\right.$$

- nun sei $x \in f_0(C_n)$ & $y \in f_1(C_n)$ $\Rightarrow$ $x = \frac{1}{3} - s$ & $y = \frac{2}{3} + t$ $\quad t, s \in [0, 1/3]$

$$\Rightarrow |g_{n+1}(y) - g_{n+1}(x)| = \left(g_{n+1}(y) - g_{n+1}\left(\tfrac{2}{3}\right)\right) + \left(g_{n+1}\left(\tfrac{1}{3}\right) - g_{n+1}(x)\right) \text{ da}$$

$$\left| \begin{array}{l} g_{n+1}\left(\tfrac{1}{3}\right) = \tfrac{1}{2} \\ = g_{n+1}\left(\tfrac{2}{3}\right) \end{array} \right.$$

$$\leq \left(y - \frac{2}{3}\right)^d + \left(\frac{1}{3} - x\right)^d \quad \text{nach } (*)$$

$$\leq t^d + s^d. \qquad \left(\frac{1}{3}\right)^d = \frac{1}{2} \;!!$$

Da $y - x = s + t + \frac{1}{3}$, reicht es zu zeigen

$$\forall s, t \in [0, \tfrac{1}{3}] \quad \Delta(s,t) = \left(\tfrac{1}{3} + s + t\right)^d - s^d - t^d \geq 0$$

da $\partial_s \Delta = d\left[\left(\frac{1}{3} + s + t\right)^{d-1} - s^{d-1}\right] \leq 0$ & $\partial_t \Delta \leq 0$

analog $\Rightarrow \min \Delta = \Delta\left(\frac{1}{3}, \frac{1}{3}\right) = 0$, also $\Delta \geq 0$.

das war die entscheidende Abschätzung

d) $n = 0$ $\Rightarrow |g_{n+k}(x) - g_n(x)| \leq 2^{-n}$ da $\text{im}(g_\ell) \subset [0,1]$ $\forall k$ $\forall x \in C_k$

nun k fix, $k \geq 1$ & Induktion in n $\quad x \in C_{(n+1)+k} \subset C_{n+k}$

$\Rightarrow \exists i \in \{0,1\}$ $\quad x = f_i(y)$ $\quad y \in C_{n+k}$ und dann mit IA:

$$|g_{n+1}(x) - g_{n+1+k}(x)| = \frac{1}{2} |g_n(y) - g_{n+k}(y)| \leq \frac{1}{2} \cdot 2^{-n} = 2^{-(n+1)}$$

Damit sind a) ... d) gezeigt.