Anfangswertproblem für die Wellengleichung

$$u_{tt} = a^2 \Delta u \quad \text{in} \quad \mathbb{R}^{N+1};$$
$$u(x,0) = f(x), \; u_t(x,0) = g(x) \quad \text{für} \quad x \in \mathbb{R}^N$$

$$\underline{N = 1:}\; u(x,t) = \frac{f(x+at) + f(x-at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} g(y)\, dy$$

(Formel von d'Alambert)

$$\underline{N = 2:}\; u(x,t) = \frac{\partial}{\partial t} \left(\frac{1}{2\pi a} \int_{K_{at}(x)} \frac{f(y)}{\sqrt{a^2t^2 - \|x-y\|^2}}\, dy \right) + \frac{1}{2\pi a} \int_{K_{at}(x)} \frac{g(y)}{\sqrt{a^2t^2 - \|x-y\|^2}}\, dy$$

(Formel von Poisson)

$$\underline{N = 3:}\; u(x,t) = \frac{\partial}{\partial t} \left(\frac{1}{4\pi a^2 t} \int_{\partial U_{at}(x)} f(y)\, do(y) \right) + \frac{1}{4\pi a^2 t} \int_{\partial U_{at}(x)} g(y)\, do(y)$$

(Formel von Kirchhoff)