

Needs["ODE'"]

To get more information type ?ODE

**ODE[{y' == x^2 + y^2, y[1] == 1}, y, x, GraphLabel -> Automatic,
PlotSolution -> {{x, 0.5, 1.5}, PlotStyle -> {Thickness[0.01]}}]**

$$\left\{ \left\{ y \rightarrow - \left(x^2 \text{BesselJ} \left[-\frac{5}{4}, \frac{x^2}{2} \right] \left(\text{BesselJ} \left[-\frac{3}{4}, \frac{1}{2} \right] + \text{BesselJ} \left[\frac{1}{4}, \frac{1}{2} \right] \right) + \right. \right. \right. \\ \left. x^2 \text{BesselJ} \left[-\frac{3}{4}, \frac{x^2}{2} \right] \left(-\text{BesselJ} \left[-\frac{5}{4}, \frac{1}{2} \right] - 3 \text{BesselJ} \left[-\frac{1}{4}, \frac{1}{2} \right] + \text{BesselJ} \left[\frac{3}{4}, \frac{1}{2} \right] \right) + \right. \\ \left. \left(\text{BesselJ} \left[-\frac{3}{4}, \frac{1}{2} \right] + \text{BesselJ} \left[\frac{1}{4}, \frac{1}{2} \right] \right) \left(\text{BesselJ} \left[-\frac{1}{4}, \frac{x^2}{2} \right] - x^2 \text{BesselJ} \left[\frac{3}{4}, \frac{x^2}{2} \right] \right) \right) \right\} / \\ \left(x \left(2 \text{BesselJ} \left[-\frac{1}{4}, \frac{x^2}{2} \right] \left(\text{BesselJ} \left[-\frac{3}{4}, \frac{1}{2} \right] + \text{BesselJ} \left[\frac{1}{4}, \frac{1}{2} \right] \right) + \right. \right. \\ \left. \left. \text{BesselJ} \left[\frac{1}{4}, \frac{x^2}{2} \right] \left(-\text{BesselJ} \left[-\frac{5}{4}, \frac{1}{2} \right] - 3 \text{BesselJ} \left[-\frac{1}{4}, \frac{1}{2} \right] + \text{BesselJ} \left[\frac{3}{4}, \frac{1}{2} \right] \right) \right) \right) \right\}$$

**Table[ODE[{y'[x] == x^2 + y[x]^2, y[1] == 1}, y[x],
x, Method -> Picard, Iterations -> n, GraphLabel -> Automatic,
PlotSolution -> {{x, 0.5, 1.5}}], {n, 0, 3}]**

{{{y[x] -> 1}}},

$$\left\{ \left\{ y[x] \rightarrow -\frac{1}{3} + x + \frac{x^3}{3} \right\} \right\},$$

$$\left\{ \left\{ y[x] \rightarrow \frac{97}{210} + \frac{x}{9} - \frac{x^2}{3} + \frac{2x^3}{3} - \frac{x^4}{18} + \frac{2x^5}{15} + \frac{x^7}{63} \right\} \right\},$$

$$\left\{ \left\{ y[x] \rightarrow \frac{14736703}{48648600} + \frac{9409x}{44100} + \frac{97x^2}{1890} + \frac{1997x^3}{8505} + \frac{128x^4}{945} + \frac{131x^5}{3150} - \frac{4729x^6}{85050} + \right. \right. \\ \left. \frac{23x^7}{315} - \frac{109x^8}{5880} + \frac{697x^9}{34020} - \frac{4x^{10}}{1575} + \frac{184x^{11}}{51975} - \frac{x^{12}}{6804} + \frac{4x^{13}}{12285} + \frac{x^{15}}{59535} \right\} \right\}$$

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Do[  
  Show[Graph[i], Graph[12]], {i, 17, 20}]
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